A Brief (but full)

ACCOUNT

OF THE

DOCTRINE

OF

Trigonometry,

BOTH

PLAIN and SPHERICAL.

BY

JOHN CASWELL, M. A.

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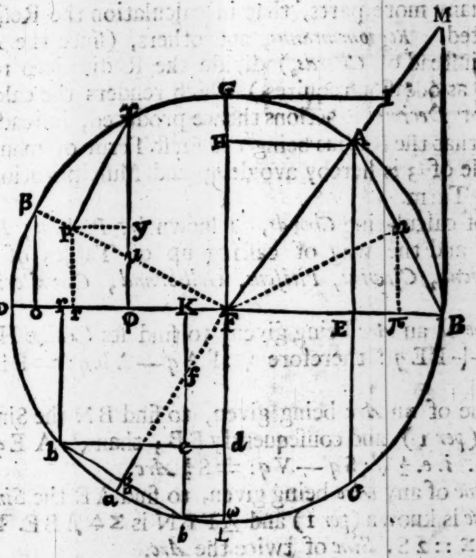
PLAIN TRIGONOMETRY.

IN this Treatise beside the Symbols, R. S. Σ. ſ. σ. T. τ. V. v. for the *Radius*, *Sine*, *Co-ſine*, *Secant*, *Co-ſecant*, *Tangent*, *Co-tangent*, *Verſed* Sine of the *Arch* and its *Complement*,) and thoſe commonly received, (+ — × = > <,) the Author makes uſe of theſe alſo.

ɜ.	Equiangular.
th:	therefore.
△.	Triangle.
⊥.	Perpendicular.
∥.	Parallel.
∠.∠∠.	Angle. Angles.
∟.	Right Angle.
Zcr.	Sum of the Legs.
Xcr.	Difference of the Legs.
Z∠∠.	Sum of the Two Angles.
X∠∠.	Difference of the Two Angles.
c. h.	(in the Column under *Giv.*) Leg. Hypothenuſe.



A *Chord* or *Subtenſe*, is a Right Line, joining the extremities of an *Arc*; as AC is the *Chord* of the Arcs ABC, ADC.

A *Right Sine* which is alſo ſingly called a *Sine*, is a Right Line drawn from one end of an *Arc*, perpendicular to the *Diameter* drawn through the other end: or it is half the *Chord* of twice the *Arc*. AE is the *Right Sine* of the *Arcs* AB, AD. The *Radius* or *Sine* of 90 degrees, is called the *whole Sine*, and is the greateſt of all *Sines*; for the *Sine* of an *Arc* greater than a *Quadrant*, is leſs than the *Radius*.

A *Verſed Sine*, called by the Ancients a *Dart* (*ſagitta*) is the Segment of the *Radius*, between the *Arc* and its *Right Sine*. EB is the *Verſed Sine* of the *Arc* AB, and ED of the *Arc* AD.

The *Secant* of an *Arc*, is a Right Line, drawn from the Center through one end of the *Arc*, till it meet with the *Tangent*, *i. e.* a Right Line touching the Circle at the nearest end of that Diameter which cuts the other end of the *Arc*. F M is the *Secant*, and B M the *Tangent* of the *Arc* AB, or of AD.

The difference of an *Arc* from a Quadrant, whether it be greater or less, is call'd the *Complement* of that *Arc*. G A is the *Complement* of the *Arcs* AB, AD; and HA is the *Sine* of that *Complement*; GI the *Tangent* of the *Complement*; FI the *Secant* of the *Complement*: Or as the *English* use to call them, HA is the *Co-sine* of the *Arc* AB; GI the *Co-tangent*; FI the *Co-secant*.

The difference of an *Arc* from a Semicircle, is called its *Supplement*.

That part of the Radius, which is betwixt the Center and its *Right Sine*, is equal to the *Co-sine*. $FE = HA$; and FO is equal to the *Co-sine* of the *Arc* DB.

If an *Arc* be greater or lesser than a Quadrant; the Sum or difference, accordingly, of the Radius and Co-sine is equal to the *Versed Sine*. $FD + HA = ED$. and $FB - HA = EB$.

Of making the Tables of Natural Sines, Tangents, *and* Secants.

IN a Triangle are 6 parts, *i. e.* 3 Sides, and 3 Angles; of these 6, any 3 being given, except the 3 Angles of a Plain Triangle, the 3 other parts may be thereby found; if, supposing the *Radius* divided into any number of equal parts, we know how many of those parts, are in the *Chord*, *Sine*, *Tangent* or *Secant* of any *Arc* proposed.

Ptolemy, with the *Ancients*, divided the *Diameter* into 120 parts, which number was chosen by reason of its many Aliquot parts. But because in this division, many *Chords* had Fractions annext, and many were Surd Roots, which created much trouble in Calculation; therefore later Mathematicians have divided the Diameter into many more parts, that in calculation the Residual Fractions may be safely neglected. *Regiomontanus*, and others, (since the *Arabians* brought in the use of *Sines* instead of *Chords*,) divide the Radius into 10000, *&c.* (adding as many Ciphers as occasion requires,) which renders the calculation much more easy, by means of *Decimal* Fractions thence produced, instead of *Sexagesimal* and *Vulgar*, and for that the Radius being the First Term of many Proportions, Division in the Rule of 3 is hereby avoided; and Multiplication, in case it be the Second or Third Term.

The Method of calculating *Chords*, is shewn by *Ptolomy*, *Regiomontanus*, *Rheticus*, *Copernicus*, and the way of casting up of Tables of *Sines* and *Tangents* is shewn by *Rheticus*, *Clavius*, *Pitiscus*, *Gellibrand*, *Cavallerius* and *Snellius*: as follows.

1. AE the *Sine* of an *Arc* being given, to find its Cosine HA.

$FAq = AEq + FEq$: therefore $\sqrt{:FAq - AEq:} = FE = HA$. or $\sqrt{:Rq - Sq:} = \Sigma$.

2. AE the Sine of an *Arc* being given, to find BN the Sine of half the *Arc*.

FE is known (*per* 1) and consequently EB; than $\sqrt{:AEq + EBq:} = AB$; and $\frac{1}{2} AB = BN$. *i. e.* $\frac{1}{2}\sqrt{:Sq - Vq:} = S \frac{1}{2}$ *Arc*.

3. BN the *Sine* of any *Arc* being given, to find AE the *Sine* of twice the *Arc*.

FN the *Co-Sine* is known (*per* 1) and $\triangle FBN$ is $\backsim \triangle ABE$. Therefore $FB . FN :: AB . AE$. *i. e.* $R . \Sigma :: 2S$. *Sine* of twice the *Arc*.

4. βO and xP, the Sines of 2 *Arcs* βD, $x\beta$ being given, to find $x\varphi$ the Sine of the *Sum* of the *Arcs*.

The *Co-sines* FO, FP, are known (by 1) but $F\beta . FP :: \beta O . Pr$; and $\triangle yPx \backsim (\triangle PUx, \backsim \triangle \varphi UF, \backsim) \triangle O\beta F$. Therefore $F\beta . FO :: xP . xy$; then $xy + Pr = x\varphi$.

5. Suppose the *Arc* $La = 30°$, and $ab = ah$, and $b\omega$ the Sine of bL, and bK its Co-sine; also bd the Sine of bL, and br its Co-sine; draw Fa cutting bh in g. Then $4ghq = bhq = beq + ehq$; Therefore, $3ghq = beq$; and $gh \times \sqrt{3} = be$; and $b\omega + gh \times \sqrt{3} = bd$. *i. e.* the Sine of an *Arc* less than 30 Degrees, adding $\sqrt{3} \times$ Sine of the defect, makes the Sine of an *Arc*, as much exceeding 30°. Therefore, if the Sines of all *Arcs* less than 30°, be known; the rest as far as 60°, may be had by one Addition, and a Multiplication into $\sqrt{3}$.

6. $\triangle gh$

6. $\Delta\, ghf \Sigma \,\Delta\, heb$, but $\angle hfg = g$FL $= 30°$; therefore $\angle hbe = 30°$. And suppoſing a Circle on the Center *g* deſcrib'd through *heb*; *be* the Chord of 60° will be $= bg$ the Radius. Therefore $rb + bg =$ K*h*: *i.e.* the Sine of any Arc *b* D leſs than 60°, adding the Sine of the Defect *ba*, makes K *h* the Sine of an Arc ſo much exceeding 60°. Conſequently the Sines of all Arcs leſs than 60° being known, the reſt to 90°, may be found by one Addition. For inſtance, the Sine of $58° +$ S $2° =$ S $62°$.

The Radius is equal to the Chord of 60°, and $\frac{1}{2}$ R $=$ S 30°, then (by 2*d*) are known the Sines of the halves 15°. 7°, 30′. 3°, 45′. 1°, 52′, 30″. 56′, 15″. 28′, 7″, 30‴. 14′, 3″, 45‴. 7′, 1″, 52‴, 30⁗. 3′, 30″, 56‴, 15⁗. 1′, 45″, 28‴, 7⁗, 30′′′′′. 52″, 44‴, 3⁗, 45′′′′′. So that by 12 Diviſions, we come to Sines which have the ſame ſenſible proportion as their Arcs; for the laſt Sine ſave one, is double of the laſt Sine to all ſenſe, as one Arc is double of the other. But $1800 \times 1' = 30° = 2048 \times$ 52″, 44‴, 3⁗, 45′′′′′. Therefore 1800. 2048 :: Arc 52″, 44‴, 3⁗, 45′′′′′. Arc 1′ :: S 52″, 44‴, 3⁗, 45′′′′′. S 1′.

And thus by a continued Biſection of Arcs, the Sine of one Minute being had (by 3*d*) is found the Sine of 2′, then (by 4*th*) the Sine of 3′, and ſo to 30°; then (by 5*th*) to 60°, and (by 6*th*) the reſt to 90°.

Gellibrand and *Vieta* find the Sines by the *Analyſis* of *Angular Sections*; inſtead of which *Pitiſcus* and others uſe the *Rule* of *falſe*.

Having the Sines, we may find the *Tangents* and *Secants* by the following Proportions.

FE.FB :: EA.BM, and FE.FB :: FA.FM. and BM.BF :: FG.GI. *i.e.* Σ.R :: S.T, and Σ.R :: R.ſ. and T.R :: R.τ.

LEMMA I.

EB.BN :: AB.FB :: ($\frac{1}{2}$AB) BN.$\frac{1}{2}$FB. *i.e.* V.S$\frac{1}{2}$Arc :: S$\frac{1}{2}$ Arc.$\frac{1}{2}$R, and therefore $\frac{1}{2}$VR $=$ S$q\frac{1}{2}$ Arc.

FN:FN FB

LEMMA II.

$\frac{1}{2}$R$v = \Sigma q \frac{1}{2}$ Arc. For draw $n\pi \perp$ FB. th: F$\pi =$ FE $+$ (Eπ) $\frac{1}{2}$EB $= \frac{1}{2}$ED. but F$\pi \times$ FB $=$ FNq.

LEMMA III.

The *Tangents* of 2 Arcs A, B, are reciprocally proportional to their *Co-tangents*. For T, A.R :: R.τ, A. and T, B.R :: R.τ, B. Therefore T, A × τ, A $=$ R × R $=$ T, B × τ B: Therefore T, A.T, B :: τ, B.τ A.

LEMMA IV.

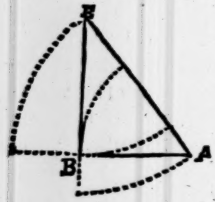
The Co-ſines of 2 Arcs A, B, are reciprocally proportional to their Secants. For Σ, A.R :: R.ſ, A. And Σ, B.R :: R.ſ, B. Therefore Σ, A.Σ, B :: ſ, B.ſ, A.

Plain and Spheric Trigonometry, are uſually reſolved into 4 fundamental Theorems, call'd Axioms.

The Firſt AXIOM.

In a Right-angled Triangle, if one Leg of the Right Angle be made the Radius of a Circle; the Hypothenuſe will be the Secant of the Adjacent Angle, and the other Leg will be the Tangent of that Angle. But if the Hypothenuſe be Radius, the 2 Legs will be Sines of the oppoſite Angles; as is manifeſt by the following Figure.

In the following Proportions, I ſuppoſe that 2 Lines being eſtimated in parts of any meaſure for example in parts of the Table are proportional to themſelves



ſelves reckon'd according to any other meaſure; ſo AB reckon'd as Radius of parts 100000, is to BE Tangent of the Angle A 30°, of Tabular parts 57735 :: ſo the ſame AB of 213 feet to BE 123 feet almoſt.

Note alſo, that becauſe few Books have Tables of Logarithmic Secants, I have declined their uſe for the moſt part, which had I admitted, I might eaſily have varied the following proportions both of Plain and Spheric Trigonometry many other ways, as *Clavius* has done. But I have regarded the giving not ſo much a multitude, as of one good Solution to each caſe; and ſuch I count that proportion to be, which has the Radius in the firſt place, for which end I have given an inſtance or Two of the Secants.

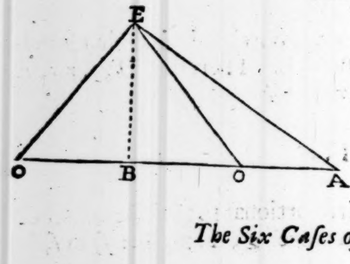
In a Right-angled Triangle, one Acute Angle is the Reſidue of the other Acute to 90°, conſequently one being known, the other is known: And in an Oblique-Angled Triangle, Two Angles being known, the Third is alſo known, as being the Supplement or Reſidue of the Sum of the other 2 to 180°.

The Seven Caſes of Right Angled Triangles.

See the laſt Figure.

Given	Requir'd	Proportions	giv.	req.
AB. ∠∠	BE	R . AB :: T, A . BE	*c*. ∠∠	*c*
AB. ∠∠	AE	S, E . AB :: R . AE or, R . AB :: ſ, A . AE	*c*. ∠∠	*h*
AB. AE	∠∠	AE . R :: AB . S, E	*c*. *h*	∠∠
AB. AE	BE	AE . R :: AB . S, E, then R . T, A :: AB . BE. or √:AE+AB:×:AE—AB: = BE	*c*. *h*.	*c*
AB. BE	∠∠	AB . BE :: R . T, A.	*cc*	∠∠
AB. BE	AE	AB . BE :: R . T, A, then S, A . R :: BE . AE.	*cc*	*h*
AE. ∠∠	AB	R . S, E :: AE . AB.	*h*.∠∠	*c*

The Second AXIOM.



In any Triangle OEA, the Sides are proportional to the *Sines* of the oppoſite Angles. For OE . R :: BE . S, O, and AE . R :: BE . S, A. Therefore OE . AE :: S, A . S, O.

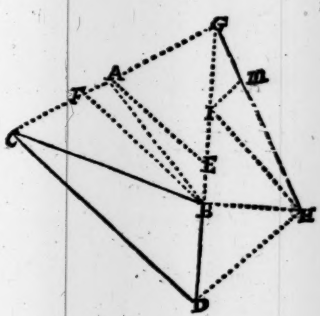
The Six Caſes of Oblique-angled Triangles.

OE . AE . A.	O	OE . AE :: S, A . S, O.	2*l*.∠*op*	∠*op*
		Note here, that whereas any Arc has the ſame *Sine*, *Tangent*, and *Secant*, with its *Supplement*; the Angle O is ſo ambiguous: *i. e.* Whether 'tis Acute or Obtuſe, cannot be diſcerned, from the Three things here given, therefore its kind muſt be found from ſome other Circumſtance of the Queſtion.		
OE . AE . A	AO	OE . AE :: S, A . S, O S, A . OE :: S, E . AO — Whence the Angle E will be alſo known, then	2*l*.∠*op*	*l*
A . O . AE	OE	S, O . AE :: S, A . OE or	2∠.*lop* 3∠.*l*	*l*.*op* *l*

The

The Third AXIOM.

The Sum of the Legs of an Angle DBC, is to the difference of the Legs :: as the Tangent of half the sum of the opposite Angles, is to the Tangent of half their difference *Dem.* producing DB, take BG=BC, and and divide DG equally in E, and GC in A. Therefore BA is ⊥ GC, and AE ∥ CD, and ∠ABC = $\frac{1}{2}$GBC = $\frac{1}{2}$Z∠∠*op.* Draw BF ∥ DC; therefore ∠FBC = altern ∠BCD. And if from ∠ABC the $\frac{1}{2}$ Sum of the Angles BCD, BDC, you take the Lesser CBF, there will remain ∠ABF the $\frac{1}{2}$ difference of the same BCD, BDC; and if from the $\frac{1}{2}$ Sum of the Legs ED, you take the lesser Leg BD, the Residue EB will be the $\frac{1}{2}$ difference of the Legs. But putting AB Radius, AC is the Tangent of ∠ABC, and AF is Tangent of ∠ABF; therefore Z*cr*. X*cr* :: ($\frac{1}{2}$ Z*cr*. $\frac{1}{2}$ X*cr* :: ED . EB :: AC . AF ::) T, $\frac{1}{2}$ Z∠∠*op*. T, $\frac{1}{2}$ X∠∠*op*.



B. BC. BD. *i.e.* ∠.2 *lspr.*	C.D.	BC + BD . BC − BD :: T, $\frac{1}{2}$ Z∠∠*op*. T, $\frac{1}{2}$ X∠∠*op*. But $\frac{1}{2}$ Z + $\frac{1}{2}$ X = greater ∠D, and $\frac{1}{2}$ Z − $\frac{1}{2}$ X = lesser ∠C.
BC. BD. B *i.e.* ∠.2 *lspr.*	CD	Find the Angles C, D by the last, then S, D . BC :: S, B . CD . or S, C . BD :: S, B . CD.

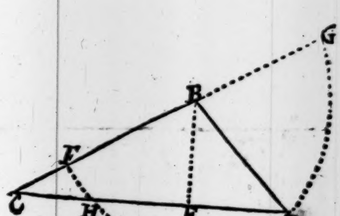
Having an Angle CBD, and the Logarithms of the Legs CB, BD, to find the other 2 Angles, which is a frequent case in Astronomy. *See the last Figure.*

The lesser Leg BD, is to the greater BC :: as the Radius to the Tangent of an Arc, from which taking 45 Degrees, as the Radius to the Tangent of the remaining Arc :: so the Tangent of the $\frac{1}{2}$ Sum of C, D, to the Tangent of their $\frac{1}{2}$ difference.

Dem. Draw BH ⊥ and = BD = BI, and IM ⊥ IH. then BH . BG :: R . T, ∠BHG, whence taking ∠BHI . R . T, ∠IHG :: IH . IM :: DH . IM :: DG . IG :: [by 3*d* Axiom] T, $\frac{1}{2}$ Z∠∠*op*. T, $\frac{1}{2}$ X∠∠*op*.

The Fourth AXIOM.

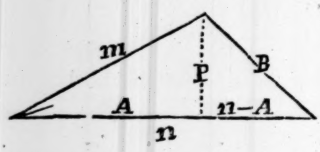
Having drawn a Perpendicular from an Angle to its opposite Base, the Base CD will be to the Sum of the Legs CG :: difference of the Legs CF, to the difference of the Segments of the Base CH.



3 Sides BC. BD. CD.	3∠	CD . CB + BD :: CB − BD . CH. then $\frac{1}{2}$CD + $\frac{1}{2}$CH = CE. and $\frac{1}{2}$CD − $\frac{1}{2}$CH = ED. and then CB . R :: CE . S̸, C, and BD . R :: ED . S̸, D.

This

This last case may be better solved by Four other Theorems. For the analysis and demonstration whereof, suppose m, n, the legs of the Angle required; B its Base, A and N $\backsim$ A, the Segments of the Base made by P, a Perpendicular let fall from another Angle, whether it fall within or without the Triangle $z = m + n$, $x = m - n$, $\mathfrak{z} = \frac{1}{2}$ Sum of m, n, B. Then is $mm - AA = PP = BB - nn - AA + 2nA$. Therefore $\frac{mm + nn - BB}{2n} = A$; therefore R. Σ of the Angle :: m. $\frac{mm + nn - BB}{2n}$ :: $2mn . mm + nn - BB$. Therefore $2mn . (2mn \pm mm \pm nn \mp BB =) \frac{zz - BB}{BB - xx}$ :: R . R $\pm \Sigma = \frac{v}{V}$ :: $\frac{1}{2}$RR . $\frac{\frac{1}{2} R v}{\frac{1}{2} RV} = \frac{\Sigma q}{Sq}$ $\frac{1}{2}$ Ang. Therefore $4mn$. Rq :: $zz - BB$. $\Sigma q \frac{1}{2}$ Ang :: $BB - xx$. S$q \frac{1}{2}$ Ang. and $zz - BB . BB - xx$ :: $\Sigma q \frac{1}{2}$ Ang. S$q \frac{1}{2}$ Ang. :: Rq. T$q \frac{1}{2}$ Ang. :: τq. Rq.



1. $4mn$. Z + B : x : Z − B :: Rq . $\Sigma q \frac{1}{2}$ Ang. or $mn . \mathfrak{z} : \times : \mathfrak{z} - B$ :: Rq . Σq. $\frac{1}{2}$ Ang.

2. $4mn$. B + x : × : B − X :: Rq . S$q \frac{1}{2}$ Ang. or $mn . \mathfrak{z} - m : \times : \mathfrak{z} - n$ :: Rq. S$q \frac{1}{2}$ Ang.

3. Z + B : × : Z − B . B + X : × : B − X :: Rq . T$q \frac{1}{2}$ Ang. or $\mathfrak{z} \times : \mathfrak{z} - B . \mathfrak{z} - m : \times : \mathfrak{z} - n$ :: Rq . T$q \frac{1}{2}$ Ang.

4. B + X : × : B − X . Z + B : × : Z − B :: Rq . $\tau q \frac{1}{2}$ Ang. or $\mathfrak{z} - m \times \mathfrak{z} - n . \mathfrak{z} : \times : \mathfrak{z} - B$ :: Rq. $\tau q \frac{1}{2}$ Ang.

Because $zz - xx = 4mn$, if you have an Angle given, with its Base, and the Sum or difference of its Legs, you will have by these Theorems the Square of the Difference or Sum, and so both the Sum and Difference, and consequently the very Legs.

If instead of Tables, you would work with a Sector, or Gunters line, or other proportional instrument; the first Theorem $2mn . BB - xx$ :: R . V, is to be resolv'd into 2 Proportions. $2m$. B + x :: B − x . G . then n . G :: R . V . or into these, B + x . $2n$ :: m . H . then, H . B − x :: R . V.

SPHE-

SPHERIC TRIGONOMETRY.

A Spheric Triangle is that which is contained between the Arcs of 3 great Circles of the Sphere.

A Spheric Angle is the ſame with the mutual aperture or inclination of the planes of thoſe 2 Circles which conſtitute the Angle.

Affections of Spheric Triangles.

1. When a Circle falls on another Circle, the Sum of the 2 Angles made thereby is $= 2 \lrcorner$.

2. When a Circle croſſes a Circle, the Vertical Angles made thereby are mutually equal.

3. The greater Angle is oppos'd to the greater ſide.

4. An iſoſceles Triangle has its 2 Angles at the Baſe mutually equal, and on the contrary, if a Triangle has 2 Angles equal, it has 2 ſides equal.

5. Two Triangles mutually equilateral, are alſo Equiangular one to the other.

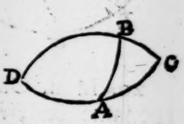
Theſe 5 proprieties, with the 2 next are common to plain Triangles, and have a like demonſtration.

6. If there be 2 Triangles, and in each, one Angle and the 2 Sides including reſpectively equal; or if one Side and the 2 Angles adjacent be ſeverally equal; then the 2 Triangles are equal; for if laid one upon another, they will agree.

7. Two Sides of a Triangle are bigger than one: For the Arc of a great Circle is the ſhorteſt diſtance betwixt 2 points on the Surface of a Sphere; as a Streight Line is betwixt 2 points in a plain.

8. All great circles cut each other into 2 equal parts; for their common Section is a Diameter of the Sphere, and conſequently the 2 Sections of the Peripherys of 2 great Circles are at a Semicircles diſtance.

Hence it follows that every Side of a Spheric Triangle, is leſs than a Semicircle. DB is leſs than the Semicircle DC.



9. The Oppoſite Angles at the Sections of 2 Circles are equal, $\angle D = \angle C$, for the ſame Planes conſtitute both Angles.

10. In any Spheric Triangle, if the Sum of the Legs of an Angle be $> = <$ (greater, equal, leſs than a) Semicircle, the intern Angle at the Baſe is (accordingly) $> = <$ outward oppoſite, and conſequently the Sum of the 2 intern Angles at the Baſe is $> = < 2 \lrcorner$. *Dem.* If $DB + BA > = < DC$, then BA is $> = < BC$, and therefore $\angle (C)\, D > = < \angle BAC$, and $\angle D + \angle DAB > = < (\angle BAC + DAB =)\ 2 \lrcorner$.

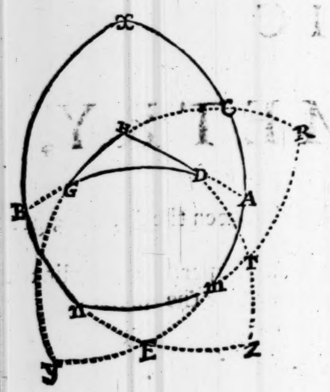
11. Coroll. In an Iſoſceles Triangle, if one of the Equal Legs is $> = <$ Quadrant, the Angle at the Baſe is $> = < \lrcorner$.

12. The Sum of the 3 Sides of a Triangle, is leſs than a Circle; for $BA < BC + AC$. *th.* $DB + DA + BA < DBC + DAC$.

13. If from the point of an Angle as a Pole, you deſcribe a great Circle, or which is the ſame, if you deſcribe a Circle at the diſtance of 90 Degrees from the point, the Arc of this Circle intercepted between the Legs of the Angle, is the meaſure of the Angle.

14. The

14. The poles of the Sides of any Triangle G H D, constitute another Triangle *n x m*, which we may call supplemental to the △GHD; for the Supplements of the Angles and Sides of △*n x m*, are equal to the Sides and Angles of the Triangle GHD.



Dem. From the points GHD as Poles, describe 3 great Circles, *x* A Y, R T *m n*, *x* B *n* Z; then is Y*m* = Quadrant = A*x*; because *m* is the Pole of HGY, and *x* or E the Pole of GA, therefore *m x* = AY = Supplement of CA = ∠HGD. and Z*n* Quadrant = BX, therefore *n x* = BZ = *Suppl.* ∠HDG. and *n*T = Quadrant = *m*R. Therefore *n m* = TR = Supplement of ∠DHG.

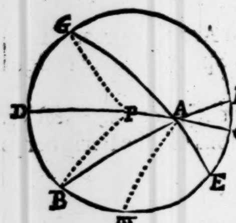
Note that the Triangle *n* E *m* constituted between the 3 next poles, has its 3 Sides and Angles = Angles and Sides of △GHD, save that the greatest Side *n m* is the Supplement of the greatest ∠H, and ∠E of the Side GD.

15. Any Angle of a Triangle with the difference of the other 2, is < 2 ⌋. For *x n* < *x m* + *m n*. *i.e.* 2 ⌋ — D < 2 ⌋ — G + 2 ⌋ — H. Therefore G + H — D < 2 ⌋.

16. If 2 Triangles are mutually æquiangular, they are also mutually equilateral; for because they are Equiangular, their Supplemental Triangles are equilateral (by 14*th*) and therefore Equiangular (by 5*th*) and therefore the proposed Triangles are Equilateral (by 14*th*.)

17. The 3 Angles of every Triangle are > 2 ⌋, and < 6 ⌋. For *n x* + *x m* + *n m* < 4 ⌋ (by the 12*th*,) *i.e.* 6 ⌋ — D — G — H < 4 ⌋. *i.e.* 2 ⌋ < D + G + H. Secondly, The Sum of the Intern Angles, is less than the Sum of the Intern and Extern both, which in all, make but 6 ⌋.

18. Of several Arcs of great Circles falling from the same point of the Spheres Surface on another Circle, the greatest is that which passes through the Pole of the Circle, and the next to this, is greater than that which is farther off. For suppose P the Pole of the Circle C ϖ D, and ϖ the Pole of DPC; then is AD > AB > AE > AC; and the Arc B ϖ C > BP > BD.



19. A great Circle passing through the Poles of another great Circle cuts it at Right Angles; and on the contrary, if it cut it at Right Angles, it passes through its Poles. ∠PBD = ⌋ = PGD = PDB = ϖAC.

20. In an Oblique-angled Triangle, if the Angles at the Base are like or of the same kind, *i.e.* both Acute, or both Obtuse; the Perpendicular falls within the Triangle, and the Quadrantal Arc without: But if they be unlike, the Perpendicular falls without, and the Quadrant within. For △EAF has ∠∠E, F Acute, and the Perpendicular AC falls within, and the Quadrant Aϖ without. Also △BAG has ∠∠B, G Obtuse, and the Perpendicular AD within and the Quadrant Aϖ without. But the △BAE has ∠∠BE of different kinds, and the Perpendicular AC without, and the Quadrant Aϖ within.

Moreover by the same Figure 'tis manifest, how the Ambiguities of Right Angled Triangles may be solved, *viz.*

SOLUTIONS.

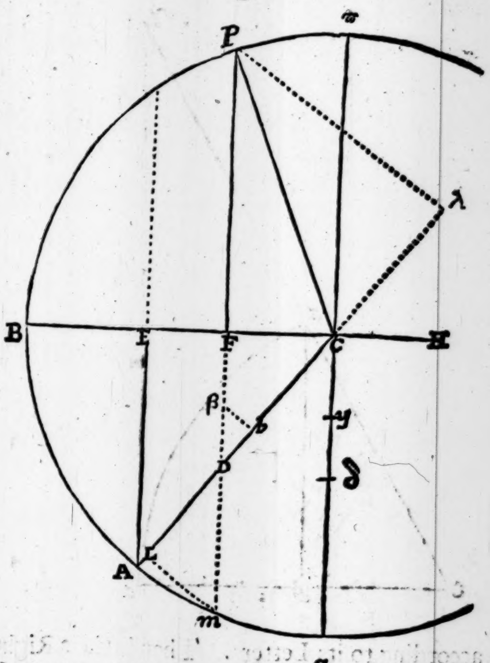
1. The Legs of the Right Angle are of the same kind with the opposite Angles. So in the △BDA, because DA > Quadrant DP, the ∠DBA > ⌋ DBP. And in △BCA, because AC < Quadrant PC, ∠CBA < ⌋ CBP.

2. If the Legs, (and consequently the Angles) are of the same or different kind; the Hypothenuse is accordingly < > Quadrant. So in the Triangles EDA, ECA, the Hypothenuse AE is < Quadrant, but in △BDA, the Hypothenuse AB is > Quadrant BP.

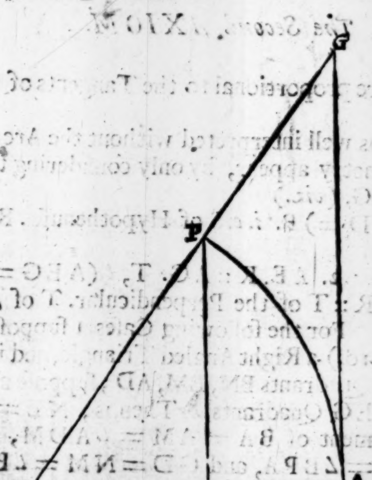
3. If

3. If the Hypothenuse is $<>$ Quadrant, either Leg with its adjacent Angle, is accordingly of the same or different kind; as follows from the 2 last.

For the viewing the Sines, Cosines and other Right Lines of Arcs which are not visible in a common Sphere: I use the Arcs of 3 great Circles of Card Past-board,



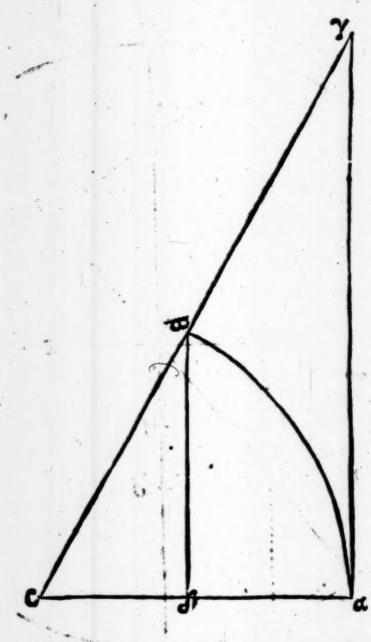
put together as in an Armillar Sphere. As suppose 2 Arcs BP, BA, and that BPH the Plane of the greater Arc were turn'd round BH, till that a Right Line falling from P Perpendicular to the Plane BAH, may fall on some point of the Line CA, suppose on D; for in that Position PAB will be a Spheric Triangle Right Angled at A, and BP the Hypothenuse, BA the Base, PA the Perpendicular Arc. And suppose PA (*Fig.2d pag.9.*) = PA of the Triangle, and fitted according



to its Letters therein; and draw AE, PF $\perp$ BC; so AE, PF, PD, will be Sines of the Arcs BA,BP,PA, and their Co-sines will will be EC,FC,DC. These things being done or conceiv'd, the 2 first Axioms of Spheric Trigonometry, will presently

appear,

appear, and also the Demonstration of the 16 Cases of Right Angled Triangles, without any other Figure or Production of Sides as is usual. However I shall in general observe the common Method, to which end let the Arc $\varpi\alpha$ (*Fig. 1. pag. 10.*) be also



fitted in the Solid according to its Letters. Then in the 2 Right Angled Spheric Triangles P B A, ϖ B α, having the same Acute Angle B at the Base,

The First AXIOM.

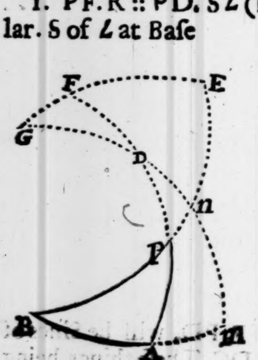
The Sines of the Hypothenuses are Proportional to the Sines of the Perpendiculars PF. PD :: ϖC. $\varpi\delta$.

The Second AXIOM.

The Sines of the Bases are proportional to the Tangents of the Perpendiculars. AE. AG :: αC. $\alpha\gamma$.

These 2 Axioms may be as well interpreted without the Arc $\varpi\alpha$, and their congruity with Plain Trigonometry appear, by only considering the plain Right-angled Triangles F D P, E A G. (*viz.*)

1. PF. R :: PD. S∠(PFD =) B. *i. e.* S of Hypothenuse. R :: S of Perpendicular. S of ∠ at Base



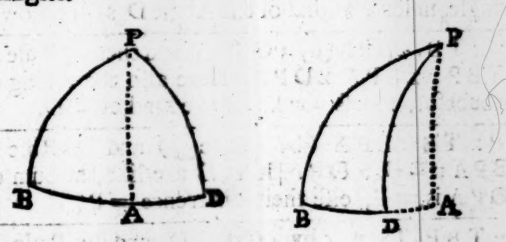
2. AE. R :: AG. T, ∠(AEG =) B. *i. e.* S of Base. R :: T of the Perpendicular. T of ∠ at the Base.

For the following Cases, I suppose BAP (*Fig. 2d pag. 10,*) a Right Angled Triangle, and its Sides produc'd to Quadrants BN, BM, AD; suppose also, PE, PF, NG and EG Quadrants. Then is, NE = BP, and Complement of BA = AM = ∠ADM, and FE = ∠FPE = ∠BPA, and GD = NM = ∠B. and the Angles at A, M, N, E, F right.

Given

Given	Req.	The Proportions for the 16 Cases of Right Angled Triangles, with the Solutions of their Ambiguities.	giv.	req.	nu.
BA. PA	BP	S,DA. S,AM :: S,DP. SPN (by 1 *Ax.*) *i. e.* R. Σ BA :: Σ PA. Σ BP. | *Sol.* 2.	*cc*	*h*	1
BA. PA	B	S,BM. S,BA :: T,MN. T,PA. (by 2*d Ax.*) *i.e.* R. SBA (:: T,B. T,PA) :: (by *Lem.* 3*d*) τ PA. τ B | *Sol.* 1.	*cc*	∠	2
BP. P.	B	SPE. SPN :: TEF. TND. (by 2*d Ax.*) *i. e.* R. Σ BP :: TP. τ B :: by 3*d Lem.* T,B. τ P | *Sol.* 1.	*h.* ∠	∠	3
BP. P.	PA	S. GE. SGF :: TEN. TFD. *i. e.* R. Σ P :: TBP. TPA. or R. Σ B :: TBP. TBA. | *Sol.* 3.	*h.* ∠	∠ *adj*	4
BP. P.	BA	R. SBP :: SP. SBA. (by *Ax.* 1) *Sol.* 1.	*h.* ∠	*c. op*	5
PA. P.	BP	SGE. SGF :: TEN. TFD :: τ FD. τ EN. *i.e.* R. Σ P :: τ PA. τ BP | *Sol.* 3.	*c.* ∠ *adj*	*h*	6
PA. P.	B	SPF. SPD :: SFE. SDN. *i. e.* R. Σ PA :: SP. Σ B. | Sol. 1.	*c.* ∠ *adj*	∠	7
BA. B.	PA	SBM. SBA :: TMN. TPA. *i. e.* R. SBA :: TB. TPA. | *Sol.* 1.	*c.* ∠ *adj*	*c*	8
PA. B.	BA	R. τ B :: (TB. R ::) TPA. SBA (by *Ax* 2) | *Ambig.*	*c.* ∠ *op*	*c*	9
PA. B.	BP	SB. SPA :: R. SBP (*Ax* 1.) *Ambig.*	*c.* ∠ *op*	*h*	10
PA. B.	P	SPD. SPF :: SDN. SFE. *i. e.* Σ PA. R :: Σ B. SP | *Ambig.*	*c.* ∠ *op*	∠	11
PA. BP	P	R. τ BP :: (TBP. R :: (by 4*th* Case) TPA. Σ P | *Sol* 3.	*c. h.*	∠ *adj*	12
PA. BP	B	SBP. R :: SPA. SB | *Sol.* 1.	*c. h.*	∠ *op*	13
PA. BP	BA	Σ PA. R :: Σ BP. Σ BA (by 1*st* Case) | *Sol.* 2.	*c. h.*	*c*	14
B. P	BP	R. τ P :: (τ P. R :: (by 3*d* Case) τ B. Σ BP | *Sol.* 2.	∠∠	*h*	15
B. P	PA	SP. R :: Σ B. Σ PA (by 11 Case) | *Sol.* 1.	∠∠	*c*	16

In Oblique-Angled Triangles, having let fall a perpendicular to make two Right-Angled Triangles.



RULE I.

The Co-sines of the Angles at the Base are proportional to the Sines of the Angles at the Vertex. For (by 7 Case of ⊥tr.)

Σ B. S∠BPA :: (Σ PA. R ::) Σ D. SDPA.

RULE II.

The Co-sines of the Sides are Proportional to the Co-sines of the Bases. For by 1 Case of ⊥tr.

Σ BA. Σ BP :: (R. Σ PA ::) Σ DA. Σ DP.

RULE

RULE III.

The Sines of the Bases are reciprocally proportional to the Tangents of the Angles at the Base. For (by *2d Ax.*)

SBA.R :: TPA.TB. and SDA.R :: TPA.TD. *th.* SBA.SDA :: TD.TB.

RULE IV.

The Tangents of the Sides are reciprocally proportional to the Co-sines of the Angles at the top. For (by 4 Case of ⌋ *tr.*)

TBP. TPA :: R. ΣBPA. and TDP. TPA :: R. ΣDPA. *th.* TBP. TDP :: ΣDPA. ΣBPA.

The Third AXIOM.

In any Triangle, the Sines of the Sides are proportional to the Sines of the Opposite Angles. For (by *1st Ax.*)

SBP.R :: SPA.SB. and SDP.R :: SPA.SD. *th.* SBP.SDP :: SD.SB.

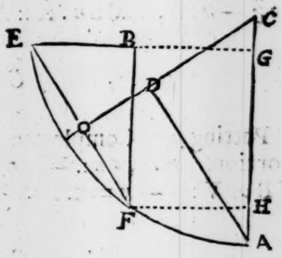
Given	Req.	The Proportions for the 12 Cases of Oblique-angled Triangles.	giv.	req.	nu.
BP.PD.B	D	SPD.SB :: SBP.SD | *Ambig.*	2 *l.* ∠ *op*	∠ *op*	1
BP.B.D	PD	SD.SBP :: SB.SPD | *Ambig.*	2 ∠ *l. op*	*l op*	2

See the two last Figures. The 8 following Cases are resolv'd by letting fall from the extream of a given Side, a Perpendicular opposite to a Given Angle. And you must observe the Addition or Subtraction both of the Segments of the Base, and Angles at the top, according as the Perpendicular falls within or without the Triangle.

BP.PD.B	BD	R. ΣB :: TBP. TBA, by 4 Case ⌋. And then by Rule 2*d*. ΣBP. ΣBA :: ΣDP. ΣDA. | But here 'tis doubtful whether the Perpendicular falls within or without the Triangle, unless the kind of the Angle D is foreknown.	2 *l.* ∠ *op*	*l*	3
BP.PD.B	P	R. ΣBP :: TB. τBPA (by 3 Case ⌋) and then by Rule 4. TDP. TBP :: ΣBPA. ΣDPA. | Here also the falling of PA is doubtful, unless you know the kind of ∠ D.	2 *l.* ∠ *op*	∠ ↄ*pr*	4
BP.B.D	P	R. ΣBP :: TB. τBPA (by 3 Case ⌋) and by Rule 1. ΣB. SBPA :: ΣD. SDPA | If B, D are like, the Sum of BPA, DPA is = P, else their difference = P.	2 ∠.*l.op*	∠	5
BP.B.D	BD	R. ΣB :: TBP. TBA (by 4 Case ⌋) and by Rule 3. TD. TB :: SBA. SDA. | If B is like D, the Sum of BA, AD is = BD, else the difference.	2 ∠.*l.op*	*l.* ↄ*pr*	6
B.P.BP	D	R. ΣBP :: TB. τBPA. (by 3 Case ⌋) then by Rule 1, SBPA. SDPA :: ΣB. ΣD. | If BPA is greater than BPD; and also B $\frac{\text{Acute}}{\text{Obtuse}}$, D is $\frac{\text{Obtuse}}{\text{Acute}}$. But if BPA is > BPD and B $\frac{\text{Acute}}{\text{Obtuse}}$, D is $\frac{\text{Acute}}{\text{Obtuse}}$.	2 ∠. *l*ↄ*pr*	*l.*	7
B.P.BP	DP	R. ΣBP :: TB. τBPA. (by 3 Case ⌋.) then by Rule 4. ΣDPA. ΣBPA :: TBP. TDP | If DPA is like, unlike B: DP is less greater than a Quadrant.	2 ∠. *l*ↄ*pr*	*l.* ↄ*p*	8

BP.BD.B	DP	R . ΣB :: TBP . TBA (by 4 Case ⊥.) then by Rule 2. ΣBA . ΣBP :: ΣDA . ΣDP | If DA is like, unlike (PA) ∠B: then PD $> <$ 90°.	2 l.∠ opr	l	9
BP.BD.B	D	R . ΣB :: TBP . TBA (by 4 Case ⊥.) then by Rule 3. SDA . SBA :: TB . TD | If BA $< >$ BD: D will be like, unlike B.	2 l.∠ opr	∠	10

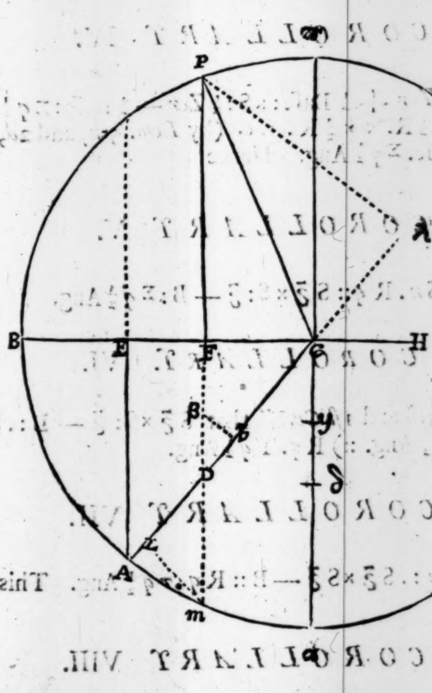
LEMMA V.



The difference of the Versed Sines of 2 Arcs × $\frac{1}{2}$R = S $\frac{1}{2}$ Sum of the Arcs Multiplied by the Sine of $\frac{1}{2}$ difference of the Arcs. For suppose AF, AE, the 2 Arcs, the difference of the Versed Sines is AG — AH = EB, and S $\frac{1}{2}$ X Arcs = FO, and S $\frac{1}{2}$ Z Arcs = AD. But △ACD ≾ △EFB. *th*: AC . AD :: FE . FB. *th*: $\frac{1}{2}$ AC × FB = AD × FO.

The Fourth AXIOM.

A Rectangle or Product of the Sines of the Legs. R q :: difference of the Versed Sines of the Base and of the difference of the Legs, to the Versed Sine of the Vertical Angle. *Dem.* Resume the foresaid Circles of Pastboard, and suppose there B the Angle requir'd, BA, BP (= BM) its Legs, PA the Base to be any



way Oblique not Perpendicular to the Plane BA as before. Let fall Pβ ⊥ βm, and γy ⊥ cα, and Pb ⊥ bA, and mL ⊥ cA. *th*: βb ⊥ cA, and bL = bA — LA = V Base — V *differ.* But AE . AG :: bL . βm. and mF . αc :: βm . yα. *th*: Multiplying the Correspondent Terms of both Proportions, AE × MF . AG × αC :: (bL × βm . βm × yα ::) bL . yα.

Case

Case II. The 3 Sides of any Spheric Triangle, being given to find an Angle.
▯ Sines *cr*. R*q* :: S: ½ Base + ½ diff. *cr*. × S: ½ Base — ½ diff. *cr*. S*q* ½ Angle.

Dem. A E × *m* F. R*q* :: (by *4th Ax.*) *b* L . *ya* :: *b* L × ½ R . *ya* × ½ R. *i.e.* (by *Lemma 5th* and *1st.*) :: S: ½ Base + ½ diff *cr* : S: ½ Base — ½ diff *cr*. S*q* ½ Angle.

COROLLARY I.

Suppose *m*, *n* the Legs of the Angle, B the Base, Z = *m* + *n*; *x* = *m* — *n*. ℨ = ½ Sum of *m*, *n*, B. *th.* ▯ Sine *cr*. R*q* :: S: ℨ — *m* : × S: ℨ — *n*. S*q* ½ Angle.

Vlacq. prescribes this following, which comes to the same. S*m*. S ℨ — *m* :: S: ℨ — *n*. A. and S*n*. R :: A. Q. then R × Q = S*q* ½ Angle.

COROLLARY II.

Putting μ = Complement of *m* *Gunter* resolves the 11*th* Case by this single Proportion. S*m*. σ*n* :: Σ B ∽ S: μ + *n*: V of the Angle. For S*n*) RR (= σ*n*. and V B ∽ V: *m* — *n* = Σ B ∽ Σ: *m* — *n*: = Σ B ∽ S: μ + *n*.

COROLLARY III.

Being a Proposition like the *4th* Axiom. ▯ Sines *cr*. R*q* :: V Z*cr*. — V Base. *v* Angle. For resuming the Circles of the Pastboard, reduce the Plane BP to the same Plane with B A, so that they may make one Circle, and let fall Pλ ⊥ AC. Then *b*λ = λ A. — BA: *i.e.* V Z*cr*. — V Base. But A E. A C :: *b*λ. βP. and *m* F. R :: βP. *v* Ang. *th* : A E × *m* F. R*q* :: (*b*λ × βP : βP × *v* ::) *b*λ. *v*. Hence follows

COROLLARY IV.

▯ Sine *cr*. R*q* :: S: ½ Z*cr* + ½ Base : × S: ½ Z*cr* — ½ Base : Σ*q* ½ Ang. For A E × *m* F. R*q* :: *b*λ. *v* :: *b*λ × ½ R. *v* × ½ R. *i.e.* (by *Lem. 5th*, and *2d*,) :: S: ½ Z*cr*. + ½ Base : × : S ½ Z*cr* — ½ Base. Σ*q* ½ Ang. Hence

COROLLARY V.

S*m* × S*n*. R*q* :: S ℨ × S: ℨ — B : Σ*q* ½ Ang.

COROLLARY VI.

It follows from the *5th* and *1st Coroll.* that S ℨ × S: ℨ — B : . S: ℨ — *m* : × S: ℨ — *n* : :: (Σ*q* ½ Ang. S*q* ½ Ang. ::) R*q*. T*q* ½ Ang.

COROLLARY VII.

S: ℨ — *m* × S: ℨ — *n* : . S ℨ × S ℨ — B :: R*q*. τ*q* ½ Ang. This follows from the last *Corollary*.

COROLLARY VIII.

V Z — V X. Diameter :: V B — V X. V *L*. or :: V Z — V B. *v L*. Which is the practise of *Foster* with his Line of Versed Sines. *Dem.* (by *4th Ax.*) V B — V X. V *L* :: S*m* × S*n*. R*q* :: (by *3d Cor.*) V Z — V B. *v L* :: Sum of the *1st* and *5th* Terms. Sum of *2d* and *6th*. *i.e.* :: V Z — V X. Diameter.

COROL-

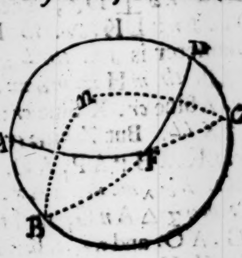
COROLLARY. IX.

(Following from 8 *Cor*,) $\frac{\Sigma Z \backsim \Sigma X}{2} \cdot R :: \Sigma B \backsim \Sigma X . V \angle .$ or. $\frac{\Sigma Z \backsim S : \mu + n.}{2}$ $R :: \Sigma B \backsim S \mu + n . V \angle .$ Which is a Theorem of frequent use with *Kepler*.

12 Case. The 3 Angles being given, to find a Side.

The Angles adjacent to the Side requir'd, call Legs, and the Angle opposite call the Base, then work as in the 11*th* Case. For such is the operation in the Supplemental Triangle, whose Angles and Sides are equal to the Supplements of the Sides and Angles of the Triangle propos'd. But Arcs and their Supplements, have the same Sines and Tangents.

In a Triangle that is Right Angled or Quadrantal, the 2 parts which are adjacent to the Right Angle or Quadrant, together with the Complements of the other 3, are called by *Neper* the 5 Circular parts. And if the 3 parts which enter the Question, (*viz.* 2 are given, and one required) have no interruption, now though a Right Angle or Quadrant come between, 'tis not counted an interruption; that part which is betwixt the other 2, is call'd the middle part, and then the other 2 are call'd Extreams adjacent or conjunct. But if there be an interruption, that part which is separate from the other 2, is called the middle part, and the other 2 Extreams opposite or disjunct. This being premised, *Neper* after a diligent view of the Solutions of all the Cases of Right Angled and Quadrantal Triangles has observed, that they all agree in one or 2 Propositions. *viz.* That Rad. × S of the middle part is = Rectangle or Product made of the Tangents of the Extreams conjunct, or to a Rectangle made of the Co-sines of the Extreams disjunct. This Proposition invented by *Neper* purely for the ease of Memory, has been since applyed in all its Cases by *Gellybrand*, *Cavallerius*, *Ursinus*, *Oughtred*, *Norwood*, *Vlacq*, *Ward* and *Wing*.



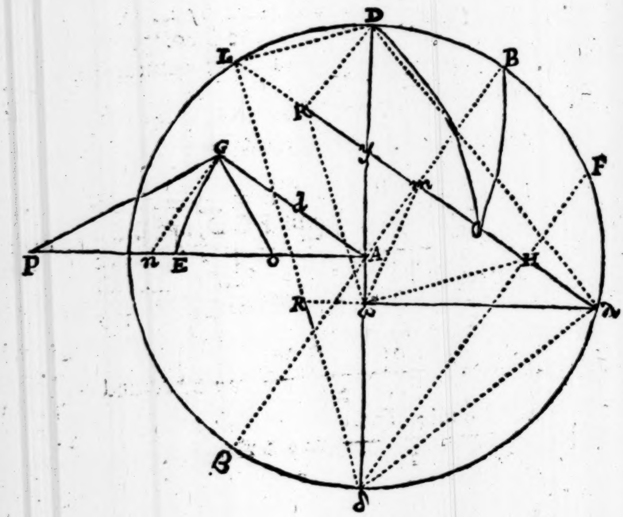
In any Spheric Triangle DAF, 2 Right Angles, are to the excess of the 3 Angles above 2 $\lrcorner$, as (G) the Surface of the Sphere, to ($4 \triangle$) 4 times the Surface of the Triangle. *Dem.* Continue the Sides to Semicircles, continue also FB, FC to Semicircles meeting again in *n*. *th.* $\triangle BnC = \triangle AFD$. But $4 \lrcorner . \angle DAF + \angle ADF + \angle AFD :: G.$ to the parts of G which are between the 3 pairs of Semicircles AFC ADC, DAB DFB, FB*n* FC*n*, which parts are manifestly $\frac{1}{2} G + 2 \triangle .$ *th.* $2 \lrcorner . \angle\angle\angle :: G . G + 4 \triangle .$ or $2 \lrcorner \angle\angle\angle - 2 \lrcorner :: G . 4 \triangle .$

Suppose ϖ the Periphery of a great Circle, and E an Arc thereof $= \angle\angle\angle - 2 \lrcorner .$ *th.* $\frac{1}{2} \varpi . E :: G . 4 \triangle .$ *th.* $2 \varpi \triangle = EG = 2 R \varpi E.$ *th.* $\triangle = RE.$

The last Proposition is I think due to *Cavallerius*, but the next following was invented by *Neper*, and is celebrated not less for its subtilty, than usefulness in resolving the last tenth Case, without letting fall a Perpendicular, or any Ambiguity.

In any Spheric Triangle DCB, 2 Sides DB, BC, and $\angle$ B included being given, to find the other Angles BC, by 2 proportions, (*viz.*)

$S \frac{1}{2} Z\, cr . S \frac{1}{2} X\, cr :: \tau \frac{1}{2}$ Vertical Angle. $T \frac{1}{2} X \angle\angle .$ and $\Sigma \frac{1}{2} Z\, cr . \Sigma \frac{1}{2} X\, cr :: \tau \frac{1}{2}$ Vert. $\angle . T \frac{1}{2} Z \angle\angle .$



Dem. Suppose A E G Poles of the Sides D B, DC, BC. *th.* the Arc A E $= \angle$ D, Arc E G $= \angle$ C, Arc A G $= 180° - \angle$ B. Suppose the Arc E O $=$ Arc E G $=$ Arc E P. Then if the points G A O E P be Stereographically projected, the Right Line A E will be $= T \frac{1}{2} \angle D$, and A G $= \tau \frac{1}{2}$ B, and A O $= T \frac{1}{2} X \angle\angle$, and A P $= T \frac{1}{2} Z \angle\angle$; and O G P will be a Semicircle described from its Pole E through G, whose Center suppose *n*. Then take Bλ $=$ BC $=$ B L, and draw the Diameter B *m* A β, and D K $\parallel$ B *m* $\parallel$ δ H. *th.* D B $= \delta\beta$, and *m* K $=$ *m* H, and F $\lambda =$ D L. Draw $\lambda\omega \perp D\delta$, then $\triangle\lambda\delta y \sim \triangle DLy$, and $DLK \sim \triangle\lambda\delta\omega$. *th.* L *y* . δ *y* :: D L . $\delta\lambda$ :: L K . $\delta\omega$. *th.* L$\delta \parallel$ Kω: But the Points $\delta\lambda$ Hω are in a Circle, whose Diameter is $\delta\lambda$. *th.* $\angle\lambda\omega H = (\angle\lambda\delta H = \angle L\delta D =) K\omega y$. *th.* $\angle K\omega H = y\omega\lambda = \perp$. *th.* *m* H $=$ *m*ω $=$ *m* K. Also λ K . L K :: ($\omega\lambda$. ωR ::) T $\angle$ D$\delta\lambda$. T $\angle$ DδL. *i. e.* Z Sine *cr*. X Sine *cr* :: T $\frac{1}{2}$ Z *cr*. T $\frac{1}{2}$ X *cr*. *th.* Z Sine $\angle\angle$. X Sine $\angle\angle$ :: T $\frac{1}{2}$ Z $\angle\angle$. T $\frac{1}{2}$ X $\angle\angle$. But Z Sine *cr*. X Sine *cr* :: (Z Sine $\angle\angle$. X Sine $\angle\angle$ ::) T $\frac{1}{2}$ Z $\angle\angle$. T $\frac{1}{2}$ X $\angle\angle$. *i. e.* λK . λH :: A P . A O. *th.* $\frac{1}{2}\lambda K + \frac{1}{2}\lambda H . \frac{1}{2}\lambda K - \frac{1}{2}\lambda H :: \frac{1}{2} AP + \frac{1}{2} AO . \frac{1}{2} AP - \frac{1}{2} AO$. *i. e.* λ*m* . *m*ω :: A *n* . *n* G. But also Angle $m\lambda\omega = \angle$ G A *n*. *th.* (by 7.6 *Eucl.*) $\triangle\lambda m\omega \sim \triangle AnG$, and *th.* $\triangle\lambda H\omega \sim \triangle AOG$, and $\triangle\lambda K\omega \sim \triangle APG$. *th.* $\lambda\omega$. λH :: A G . A O. and $\lambda\omega$. λK :: A G . A P. But Dλ . $\lambda\omega$ :: D L (L K $=$) λH. and $\delta\lambda$. $\lambda\omega$ :: δL. (L H $=$) λK. *th.* Dλ . D L :: ($\lambda\omega$. λH ::) A G . A O. and $\delta\lambda$. δL :: ($\lambda\omega$. λK ::) A G . A P. *i. e.* S $\frac{1}{2}$ Z *cr*. S $\frac{1}{2}$ X *cr* :: $\tau \frac{1}{2}$ Vert. $\angle$. T $\frac{1}{2}$ X $\angle\angle$. and $\Sigma \frac{1}{2}$ Z *cr*. $\Sigma \frac{1}{2}$ X *cr* :: $\tau \frac{1}{2}$ Vert. $\angle$. T $\frac{1}{2}$ Z $\angle\angle$.

COROLLARIES.

In any Spheric Triangle A E G.

1. T $\frac{1}{2}$ Base, T $\frac{1}{2}$ Z *cr* :: T $\frac{1}{2}$ X *cr*. T $\frac{1}{2}$ X of the Segments of the Base A G made by a Perpendicular Arc falling thereon from E.

A G . A O :: A P . A *d*.

2. T $\frac{1}{2}$ Base T $\frac{1}{2}$ Z *cr* :: $\Sigma \frac{1}{2}$ Z $\angle\angle$. $\Sigma \frac{1}{2}$ X $\angle\angle$. A G . A P :: $\delta\lambda$. δ L. For $\angle$ E A G $=$ Arc D B, and $\angle$ A G E $=$ B C.
3. T $\frac{1}{2}$ Base T $\frac{1}{2}$ X *cr* :: S $\frac{1}{2}$ Z $\angle\angle$. S $\frac{1}{2}$ X $\angle\angle$.

A G . A O :: D λ . D L.

4. If $\angle$ E G A be supposed Right, then A P $\times$ A O $=$ A G *q*. *i. e.* In a Right Angled Spheric Triangle, E G A, T : $\frac{1}{2}$ Hypot. $+ \frac{1}{2}$ Perpend. $\times$ T : $\frac{1}{2}$ Hypot. $- \frac{1}{2}$ Perpend. $=$ T *q* $\frac{1}{2}$ Base.

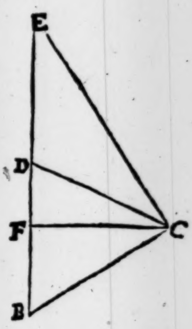
I shall

I ſhall here add 3 Theorems, ſerving to find by Calculation the Diameters or Centers of the Circles of the Sphere, in the Stereographic projection, or for the more eaſy making and proving the Tables of natural Tangents and Secants.

Draw $CF \perp BE$, and $CD = DE = DB$. *th.*

1. $\angle DCE = \frac{1}{2} FDC = \frac{1}{2}$ Complement of FCD; but $FD + CD = FE$. *i.e.* T of an Arc + S Arc = T Arc + $\frac{1}{2}$ Co-arc = T: $45^\circ + \frac{1}{2}$ Arc.

2. $\angle DCF + 2\,FCB = \angle B + FCB = \llcorner = DCF + 2\,DCE$. *th.* $FCB = DCE = \frac{1}{2}$ Compl. FCD, and $BCE = \llcorner$. But $DF + FB = CD$. *i.e.* T Arc + T $\frac{1}{2}$ Co-arc = ſ Arc. For example T 60° + T 15° = ſ 60° = 2 Rad. and *th.* 2 T 60° + T 15° = T 60° + 2 R = T 60° + ſ 60° = (by 1ſt Theorem) T 75°.



3. $\dfrac{FE - FB}{2} = FD$. *i.e.* $\dfrac{\tau\ \text{Arc} - T\ \text{Arc}}{2} = \tau\ 2\ \text{Arc}$. and $\dfrac{FB + FE}{2} = CD$. *i.e.* $\dfrac{\tau\ \text{Arc} + T\ \text{Arc}}{2} = \sigma\ 2\ \text{Arc}$.

F I N I S.

ERRATA.

Pag. 2. l. 44. for Sq— read Sq+. l. 54. for 4gbq = read 4gbq(4ebq) = p. 5. l. 18. for EB× read EB::